Reference Solution to Problem Set 1

October 12, 2011

1 Basic matrix operations

(3) Undefined., $\begin{bmatrix} -6 & 1 \\ 6 & -3 \\ 9 & 0 \end{bmatrix}$, $\begin{bmatrix} 6 & -1 \\ -6 & 3 \\ -9 & 0 \end{bmatrix}$**, Undefined.**

(5) $\begin{bmatrix} -48 & -2 \\ 38 & -44 \\ 67 & -15 \end{bmatrix}$, $\begin{bmatrix} 36 & 0 & 48 \\ -12 & 24 & 24 \\ 72 & 60 & -48 \end{bmatrix}$, $\begin{bmatrix} 36 & 0 & 48 \\ -12 & 24 & 24 \\ 72 & 60 & -48 \end{bmatrix}$, $\begin{bmatrix} -0.3 & -5.0 & -3.4 \\ -4.9 & 1.8 & 3.8 \\ -3.6 & 3.5 & 0.4 \end{bmatrix}$

2 Matrix multiplication

(9) $\begin{bmatrix} 337 & 8 & -160 \\ 252 & 49 & -68 \\ -308 & 52 & 233 \end{bmatrix}$, $\begin{bmatrix} 337 & 8 & -160 \\ 252 & 49 & -68 \\ -308 & 52 & 233 \end{bmatrix}$, $\begin{bmatrix} 257 & 68 & -188 \\ 232 & 97 & -96 \\ -248 & -16 & 265 \end{bmatrix}$

(11) $\begin{bmatrix} 324 & 32 & -320 \\ 244 & 38 & -322 \\ -244 & -10 & 366 \end{bmatrix}$, $\begin{bmatrix} 216 & -104 & -104 \\ 280 & -132 & -68 \\ -280 & 140 & 76 \end{bmatrix}$, $\begin{bmatrix} 7060 & 960 & -5120 \\ 7548 & 1246 & -5434 \\ -8140 & -1090 & 6150 \end{bmatrix}$, $\begin{bmatrix} 4324 & 1520 & -4816 \\ 3636 & 1242 & -4518 \\ -3700 & -1046 & 5002 \end{bmatrix}$

3 Triangular matrices

(21) Triangular are $U_1 + U_2,\ U_1U_2,\ U_1^2,\ L_1 + L_2,\ L_1L_2,\ L_1^2$

4 Solving systems of linear equations

(7) $x = 20,\ y = -3,\ z = 1$

5 Rank, vector spaces

(1) 1, $[1 - 2]$, $\begin{bmatrix} 1 & 0 & -2 \end{bmatrix}^T$

(27) 2, $\begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

(29) No

(H) All vector in $\mathbb{R}^4$ **such that** $v_1-v_2+2v_3+2v_4=0$, $2v_1+v_2+4v_3=0$, $3v_2-2v_3-4v_4=0$, $3v_1+3v_2+4v_3-2v_4=0$

6 Determinants

(c) det(A)= $\prod\limits_{1\le i\le n} x_i \prod\limits_{1\le i<j\le n}(x_i-x_j)$

7 Inverses of matrices

(9) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(13) Hint: Use the cofactor to derive the inverse of matrix

8 Linear transformation

(A) Hint: the plain $2x+4y-z=0$ **is a vector space (null space), so is the plain** $x-2y+3z=0$**; Linear transformation can map one vector space to another vector space; the dimension of domain vector space is less than or equal to the dimension of range vector space.**

9 Eigenvalues/eigenvectors

(A) B: -2, $[\,1 \quad 2 \quad 2\,]$**; 4,** $[\,1 \quad 1/2 \quad -1\,]$**; 1,** $[\,1 \quad -1 \quad 1/2\,]$**.**

(B) $B^{10}=\begin{bmatrix} 1024 & 0 & 1024 \\ 0 & 0 & 1024 \\ 1024 & 1024 & 1 \end{bmatrix}$